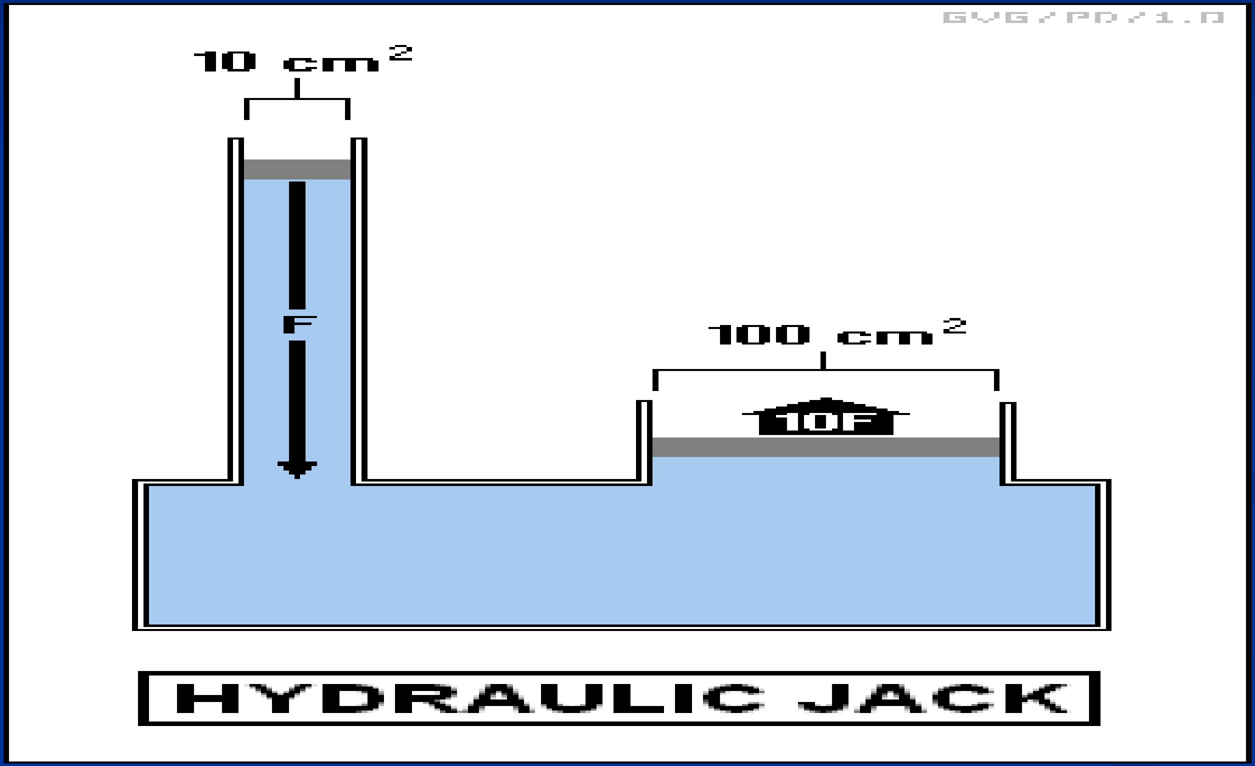
PASCAL'S PRINCIPLE



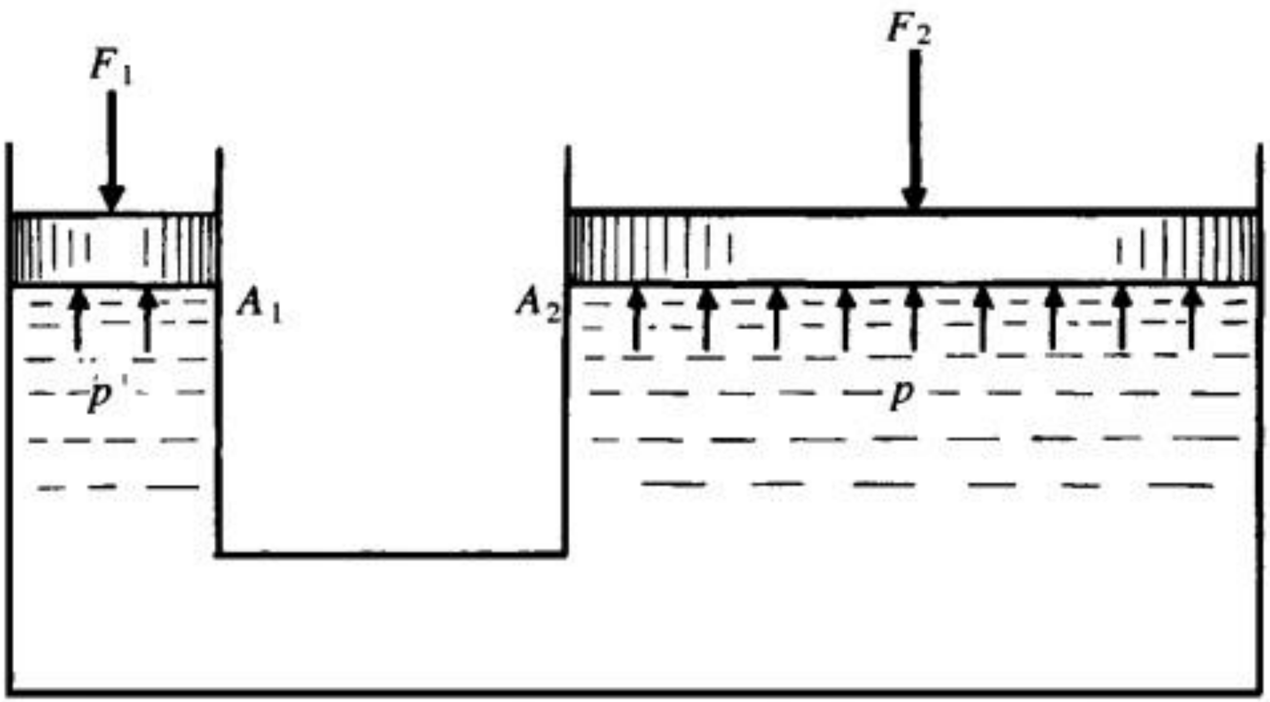


Fig. 3.3 Hydraulic press

So this device can create the large force F_2 from the small force F_1. This is the principle of the hydraulic press.

3.1.4 Pressure of fluid at rest

In general, in a fluid at rest the pressure varies according to the depth. Consider a minute column in the fluid as shown in Fig. 3.4. Assume that the sectional area is $\mathrm{d}A$ and the pressure acting upward on the bottom surface is p and the pressure acting downward on the upper surface ($\mathrm{d}z$ above the bottom surface) is $p + (\mathrm{d}p/\mathrm{d}z)\mathrm{d}z$. Then, from the balance of forces acting on the column, the following equation is obtained:

$$p\,\mathrm{d}A - \left(p + \frac{\mathrm{d}p}{\mathrm{d}z}\mathrm{d}z\right)\mathrm{d}A - \rho g\,\mathrm{d}A\,\mathrm{d}z = 0$$

or

or

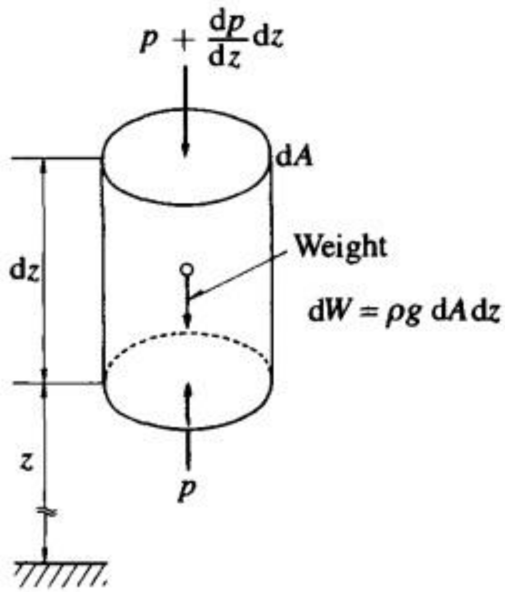


Fig. 3.4 Balance of vertical minute cylinder

$$\frac{dp}{dz} = -\rho g \tag{3.6}$$

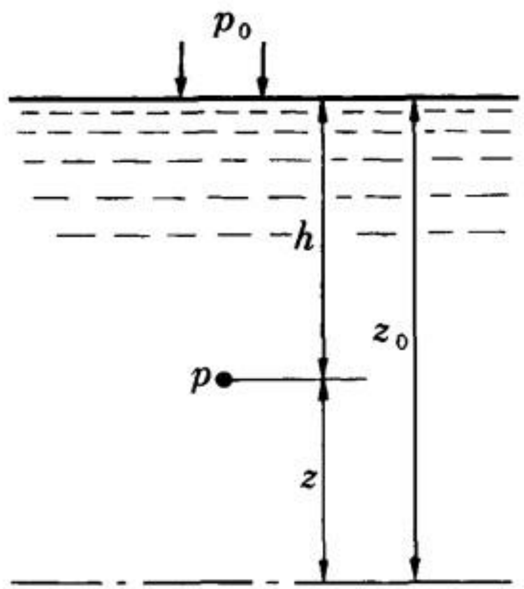
Since ρ is constant for liquid, the following equation ensues:

$$p = -\rho g \int dz = -\rho g z + c \tag{3.7}$$

When the base point is set at z_0 below the upper surface of liquid as shown in Fig. 3.5, and p_0 is the pressure acting on that surface, then $p = p_0$ whe $z = z_0$, so

$$c = p_0 + \rho g z$$

Substituting this equation into eqn (3.7),



$$p = p_0 + (z_0 - z)\rho g = p_0 + \rho g h \tag{3.8}$$

Thus it is found that the pressure inside a liquid increases in proportion to the depth.

For the case of a gas, let us study the relation between the pressure and the height of the atmosphere surrounding the earth. In this case, since the density of gas changes with pressure, it is not possible to integrate simply as in the case of a liquid. As the altitude increases, the temperature decreases. Assuming this temperature change to be polytropic, then pv^n = constant is the defining relationship.

Putting the pressure and density at $z = 0$ (sea level) as p_0 and ρ_0 respectively, then

$$\frac{p}{\rho^n} = \frac{p_0}{\rho_0^n} \tag{3.9}$$

Substituting ρ into eqn (3.6),

$$dz = -\frac{dp}{\rho g} = -\frac{1}{g}\frac{p_0^{1/n}}{\rho_0}p^{-1/n}dp = -\frac{1}{g}\frac{p_0}{\rho_0}\left(\frac{p_0}{p}\right)^{1/n} d\left(\frac{p}{p_0}\right) \tag{3.10}$$

Integrating this equation from $z = 0$ (sea level),

$$z = \int_0^z dz = \frac{1}{g}\frac{n}{n-1}\frac{p_0}{\rho_0}\left[1 - \left(\frac{p}{p_0}\right)^{(n-1)/n}\right] \tag{3.11}$$

The relation between the height and the atmospheric pressure develops into the following equation by eqn (3.11):

$$\frac{p(z)}{p_0} = \left(1 - \frac{n-1}{n}\frac{\rho_0 g}{p_0}z\right)^{n/(n-1)} \tag{3.12}$$

Also, the density is obtained as follows from eqs (3.9) and (3.12):

$$\frac{\rho(z)}{\rho_0} = \left(1 - \frac{n-1}{n}\frac{\rho_0 g}{p_0}z\right)^{1/(n-1)} \tag{3.13}$$

When the absolute temperatures at sea level and at the point of height z are T_0 and T respectively, the following equation is obtained from eqn (2.14):

$$\frac{p}{\rho T} = \frac{p_0}{\rho_0 T_0} = R \tag{3.14}$$

From eqs (3.12)–(3.14)

$$\frac{T(z)}{T_0} = 1 - \frac{n-1}{n}\frac{\rho_0 g}{p_0}z \tag{3.15}$$

From eqn (3.15)

$$\frac{dT}{dz} = -\frac{n-1}{n}\frac{\rho_0 g}{p_0}T_0 = -\frac{n-1}{n}\frac{g}{R} \tag{3.16}$$

In aeronautics, it has been agreed to make the combined values of $p_0 = 101.325\,\mathrm{kPa}$, $T_0 = 288.15\,\mathrm{K}$ and $\rho_0 = 1.225\,\mathrm{kg/m^3}$ the standard atmospheric condition at sea level.[2] The temperature decreases by 0.65°C every 100 m of height in the troposphere up to approximately 1 km high, but is constant at −50.5°C from 1 km to 10 km high. For the troposphere, from the above values for p_0, T_0 and ρ_0 in eqn (3.10), $n = 1.235$ is obtained as the polytropic index.

Pascal's Principle

When force is applied to a confined liquid, the change in pressure is transmitted equally to all parts of the fluid.

Draw a bottle of water with arrows to illustrate the regular exerted pressure. Then draw a water bottle that you squeeze.   What happens to the pressure? What happens if you open the top?

How does Pascal's Principle explain what happens if you squeeze a water bottle?

4.2 Pascal's Law

$$P_1 = P_2$$

Fluid pressure is measured in terms of the force exerted per unit area.

$$P = \frac{F}{A}$$

$$P_1 = \frac{F_1}{A_1}$$

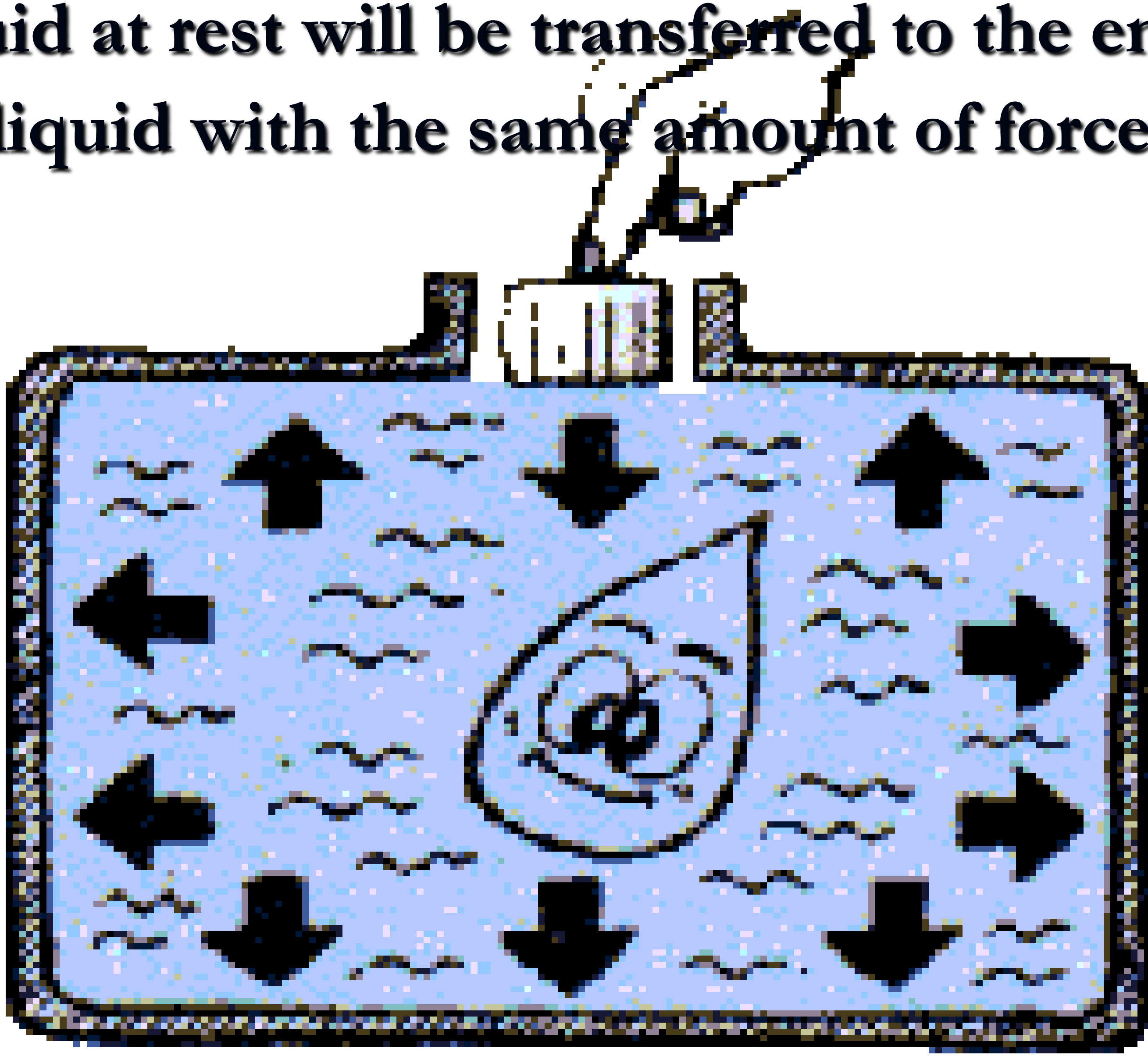
$$P_2 = \frac{F_2}{A_2}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

The values F1, A2 can be calculated using the following formula:

$$F_1 = \frac{A_1 \times F_2}{A_2}$$, and $$A_2 = \frac{A_1 \times F_2}{F_1}$$

A force applied to one section of an enclosed liquid at rest will be transferred to the entire liquid with the same amount of force.



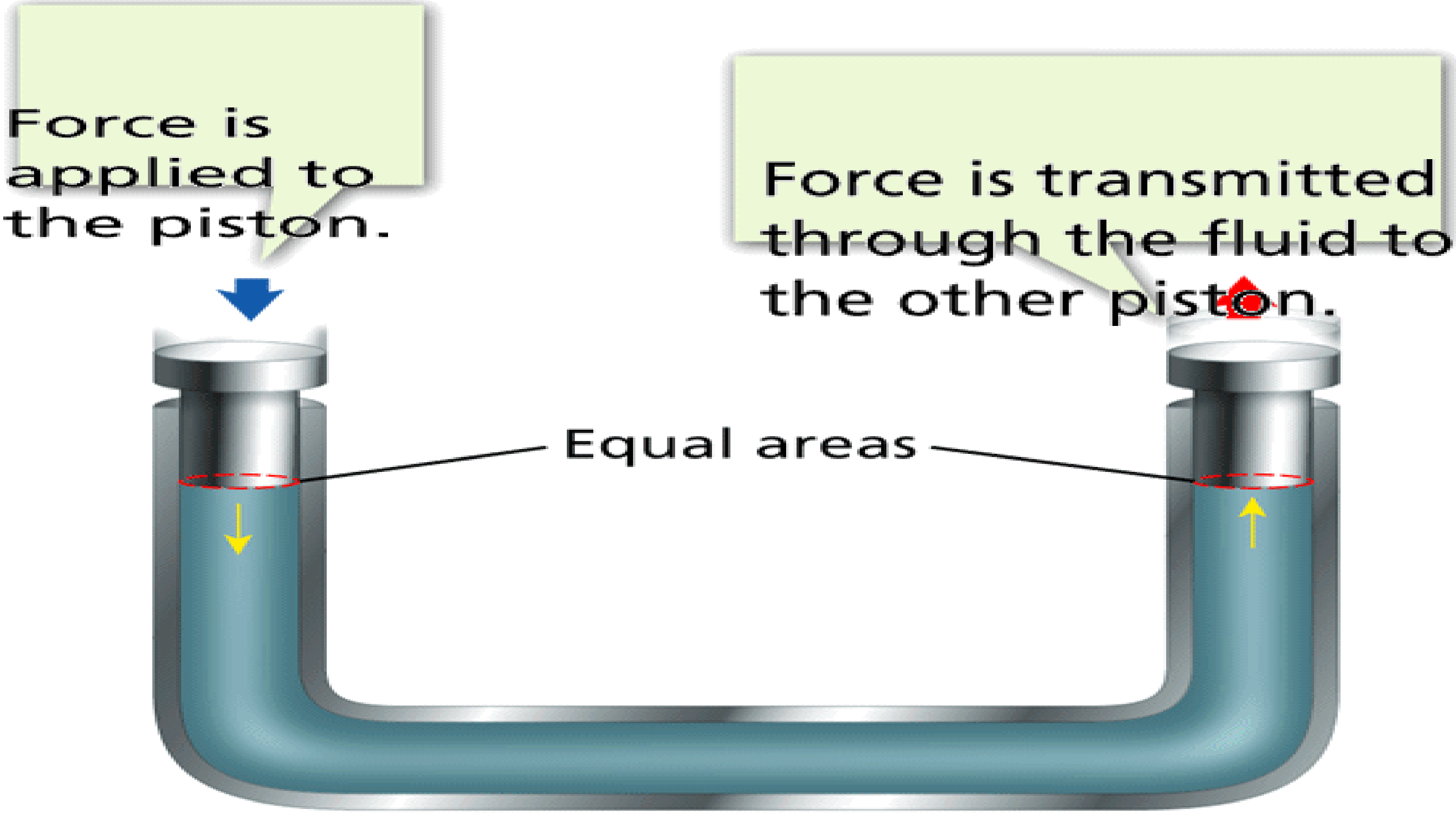
Hydraulic Systems

A force applied to one piston increases the fluid pressure throughout the fluid.

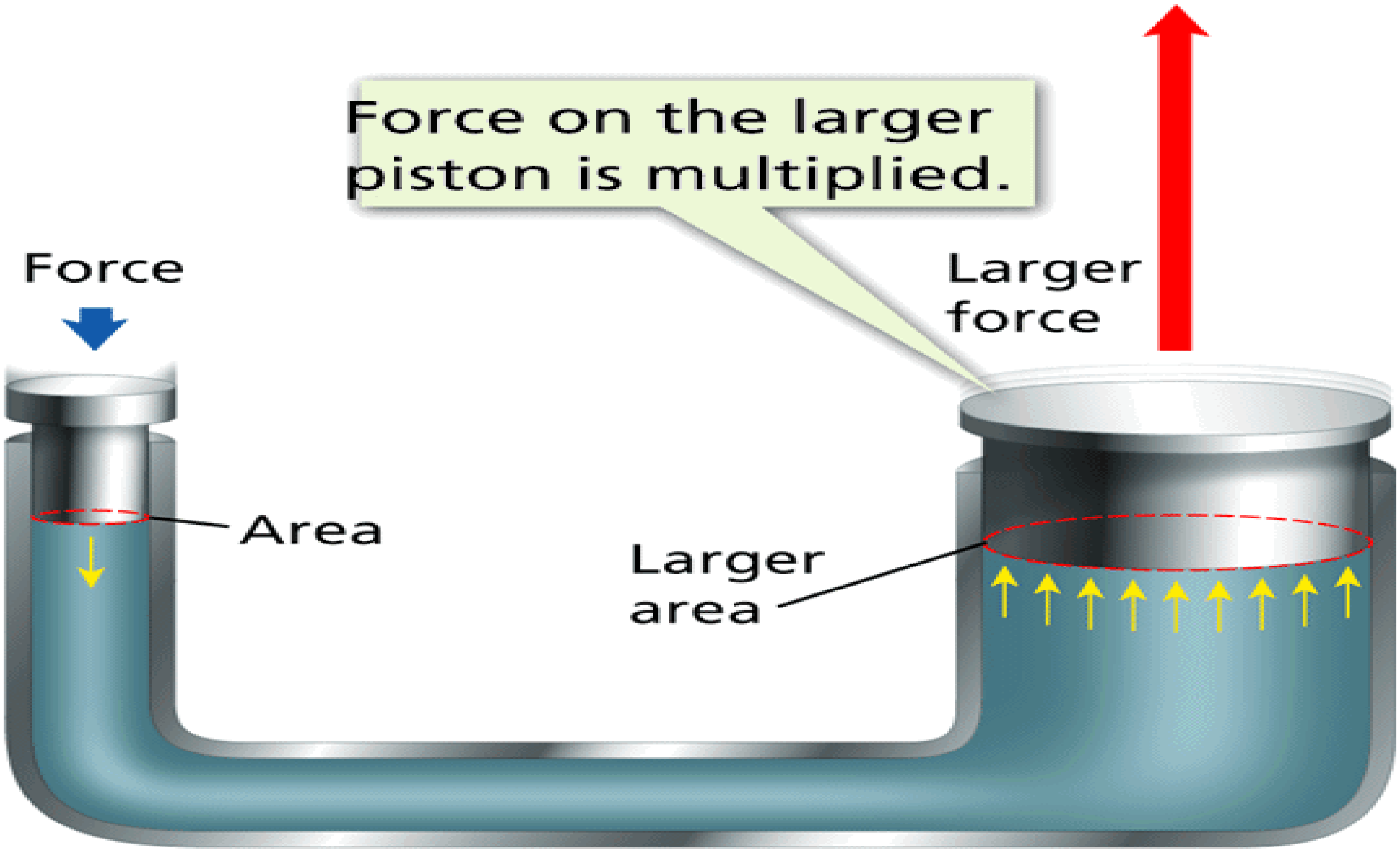
If the second piston has a larger surface area, the force is multiplied!

Pressure = Force/Area

Force/Area = Pressure = Larger Force/Larger Area

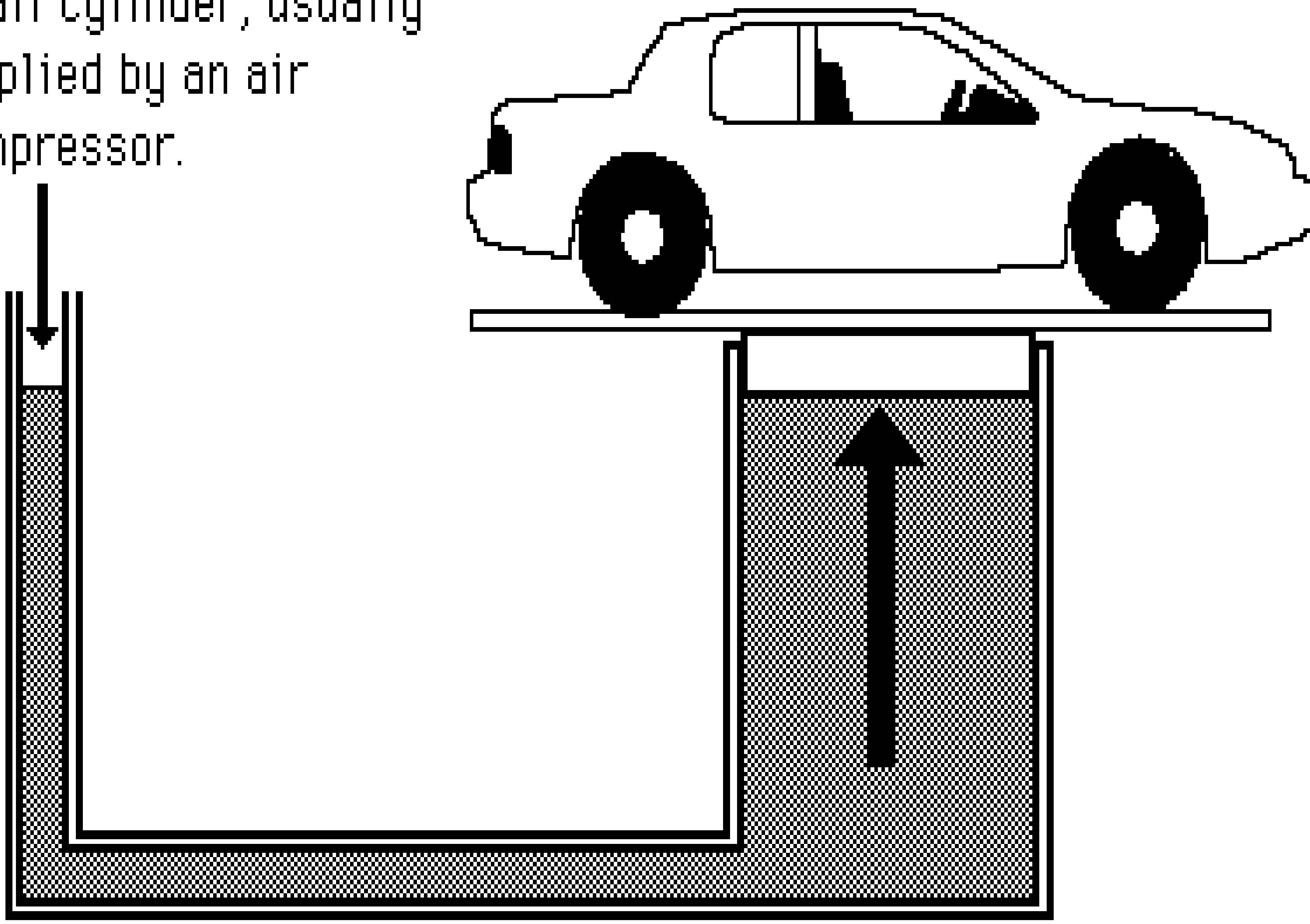


$$\frac{\text{Force}}{\text{Area}} = \text{Pressure} = \frac{\text{Force}}{\text{Area}}$$



$$\frac{\text{Force}}{\text{Area}} = \text{Pressure} = \frac{\text{Larger force}}{\text{Larger area}}$$

Pressure on fluid in small cylinder, usually supplied by an air compressor.



Example of Boyle's Law

A gas sample at a pressure of 1.23 atm has a volume of 15.8 cm^3, what will be the volume (in L) if the pressure is increased to 3.16 atm?

Do you expect volume to increase or decrease?

$\mathbf{P_{initial}}$ = 1.23 atm $\quad$ $\mathbf{P_{final}}$ = 3.16 atm
$\mathbf{V_{initial}}$ = 15.8 cm^3 $\quad$ $\mathbf{V_{final}}$ = unknown
T and **n** remain constant

$$\boxed{P_iV_i = P_fV_f}$$

$$V_i = 15.8\ cm^3 \times \frac{1\ mL}{1\ cm^3} \times \frac{1\ L}{1000 mL} = 0.0158\ L$$

$$V_f = V_i \times \frac{P_i}{P_f} = 0.0158\ L \times \frac{1.23\ atm}{3.16\ atm} = 0.00615\ L$$

Boyle's Law

- Can re-write Boyle's law to solve mathematical problems.

$$P_1 \bullet V_1 = P_2 \bullet V_2$$

- Problem
  - According to the graph what will be the volume of the gas when the pressure is 520 kPa?

P_1 = 50 kPa
V_1 = 6 L
P_2 = 520 kPa
V_2 = ? L

$$P_1 \bullet V_1 = P_2 \bullet V_2$$

$$50 \text{ kPa} \bullet 6 \text{ L} = 520 \text{ kPa} \bullet V_2$$

$$V_2 = 0.58 \text{ L}$$

Pascal's Principle Practice

A hydraulic lift lifts a **19,000 N** car. If the area of the small piston (A_1) equals **10.5 cm²** and the area of the large piston (A_2) equals **400 cm²**, what force needs to be exerted on the small piston to lift the car?

Given:

$F_2 = 19{,}000\text{N}$

$A_1 = 10.5\ \text{cm}^2$

$A_2 = 400\ \text{cm}^2$

$F_1 = ?$

Known:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Solution:

$$F_1 = \frac{(F_2)(A_1)}{A_2}$$

$$F_1 = \frac{(19{,}000\text{N})(10.5\text{cm}^2)}{400\text{cm}^2}$$

$F_1 = 500\text{N}$